Structural Condition Assessment Using Imprecise Probability

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Abstract: Investigating the condition of a structural system requires an accurate estimate of the applied load and the current condition of the structure. As such, the presence of uncertainty becomes significant; because the variations inherent in design parameters will significantly affect the reliability of the structure. Therefore, it is crucial to appropriately quantify uncertainty in the design parameters as well as perform structural reliability analyses for determining the condition of the structure and its failure potential. This paper presents a new method for condition assessment of structures, which exhibit polymorphic uncertainty in their design parameters. An imprecise probability approach is used to quantify the polymorphic uncertainty. Applying this technique to conventional methods, the reliability analysis of a structure is improved. By incorporating imprecise probability values in the reliability analysis process, bounds for the probability of failure are estimated and established. These bounds are then used as a measure for the condition assessment of the structure. A numerical example is provided to demonstrate the applicability of the developed method.

Keywords: structural dynamics, uncertainty, imprecise probability, p-box

1. Introduction

In structural engineering, the reliability and safety of a structure must be accurately assessed. Moreover, in order to achieve this reliability, the uncertainties present in both the structure and applied loads must be included in the analytical schemes. It is generally assumed in reliability analysis that all of the uncertainty is due to inherent stochasticity, known as aleatoric uncertainty, in the system and rather than due to modeling errors or faulty assumptions, known as epistemic uncertainty. This aleatoric uncertainty is accounted for using traditional theories of probability. Probabilistic methods require random variables to follow an assumed distribution, a requirement which often cannot be satisfied. One method of uncertainty modeling that avoids the shortcomings of traditional probabilistic modeling and other isomorphic uncertainty modeling methods is imprecise probability. Imprecise probability is a polymorphic uncertainty modeling method which involves setting possibilistic bounds on the cumulative distribution function describing uncertain parameters.

Numerous methods for reliability assessment of structural systems with uncertainty have been developed, the majority of which are based on traditional probability theories (isomorphic probabilistic approaches). Although theories of structural reliability are well-established, the practical application of the methods developed for reliability analysis is mathematically complicated. Moreover, the mathematical complexity increases dramatically as the number of structural components or modes of failure increases.

As a result, practicing engineers often resort to gross simplifications to overcome the complexity inherent in the general formulation of structural reliability. This leads to reliability predictions that have a significant level of error. In this work, a new method for reliability analysis of a structure using an imprecise probability approach is developed. This method offers a new direction for incorporating uncertainties in the analysis and relies on defining the uncertain parameters using imprecise probability structures. Due to its polymorphic approach, this method offers a more realistic and comprehensive yet simpler process of treating uncertainties than traditional probabilistic-based reliability analyses.

2. Review of Traditional Probabilistic Structural Reliability

2.1. Performance and Limit-State Analysis

Traditional structural reliability analyses seek to estimate the probability that a structure will be unable to withstand the applied loading, known as the probability of failure. Considering a performance function, *Z*, with multiple independent variables representing the design parameters, X_i (Ang and Tang, 2007):

$$Z = g(X_1, X_2, \ldots, X_n) \tag{1}$$

Using the performance function *Z*, the probability of failure can be defined as:

$$P_F = P(Z \leq z_o) \tag{2}$$

in which z_o is the performance limit defined as the minimum level of performance such that a structure is considered safe. Similarly, the probability of failure can be written as:

$$P_F = \int_{-\infty}^{z_o} f_Z(z)dz \tag{3}$$

in which $f_Z(z)$ is the probability density function (PDF) of the performance function *Z* in the multivariate space.

Many traditional reliability methods rely on the first order approximation, referred to as the First Order Reliability Method (FORM). This method yields sufficiently accurate results in cases where parameters X_i have small uncertainties. To enhance FORM, the Second Order Reliability Method (SORM) has also been developed. SORM also has limitations due to the increase in complexity of the analysis when the number of modes of failure increases or when there is a high level of correlation among parameters in the limit-state equation.

2.2. First Order Reliability Method (FORM)

In order to develop the general formulation for FORM, a first-order approximation on the performance function *Z* about the mean values, μ_i , of each design parameter is performed as:

$$Z = g(\mu_i) + \sum_{i=1}^{n} (X_i - \mu_i) \frac{\partial g(\mu_i)}{\partial X_i} \tag{4}$$

Assuming the variables X_i are independent, the mean, μ, and standard deviation, σ, of Z are defined as:

$$\mu = g(\mu_1, \mu_2, ..., \mu_n) \tag{5}$$

$$\sigma = \left(\sum_{i=1}^{n} c_i^2 \sigma_i^2 \right)^{1/2} \tag{6}$$

in which $c_i = \partial g(\mu_i) / \partial X_i$. FORM works well when uncertainties are small (say < 0.3).

Many performance functions exist. For example, considering a structure under a random load, *S*, and a random resistance, *R*, the performance function and performance limit can be written as:

$$Z = R - S \tag{7}$$

$$z_o = 0 \tag{8}$$

Therefore, the probability of failure is:

$$P_F = P(Z \leq 0) = \int_{-\infty}^{0} f_Z(z)dz \tag{9}$$

The mean and standard deviation of *Z* are $\mu_Z = (\mu_R - \mu_S)$ and $\sigma_Z = \sqrt{\sigma_R^2 + \sigma_S^2}$, respectively.

Assuming S and R to be random variables defined by normal probability density functions, the probability of failure is:

$$p_F = \Phi\left(\frac{z_0 - \mu_Z}{\sigma_Z} \right) \tag{10}$$

in which Φ is the standard normal cumulative distribution function. Substitution for z_o, the mean, and the standard deviation of *Z* yields:

$$p_F = \Phi\left(\frac{\mu_S - \mu_R}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right) \tag{11}$$

in which the probability of failure for the structure is evaluated based on the probabilistic values of the load and resistance.

When multiple modes of failure (m modes) are present, consideration of the two extreme cases of independence and perfect correlation among the modes allows for setting bounds on the probability of failure for the structure as:

$$\max(P_f^1, P_f^2, ..., P_f^m) \leq P_f \leq 1 - \prod_{j=1}^{m} (1 - P_f^j) \tag{12}$$

in which the lower bound is the case of perfect correlation between failure modes and the upper bound is the case of independence among failure modes. In order to ascertain the reliability of the structure, the upper bound (independence case) can be used for the reliability level of the structure.

3. Methodology

3.1. Formulation of Imprecise Probability Structural Condition Assessment

The polymorphic approach for obtaining the probability of failure enables more reliable structural condition assessment due to the consideration of polymorphic uncertainties in both the applied loads and resistance of the structure. The general algorithm for Imprecise Probability Structural Condition Assessment (IPSCA) is given below.

1. Determine the structure's modes of failure (e.g. bending, shear, deflection).
2. Determine the imprecise probability structure for the performance function for each failure mode. For each failure mode:
    a. Construct independent imprecise probability structures for the uncertain load and resistance.
    b. Perform random sampling on the CDF probability levels of uncertain load and resistance imprecise probability structures. For each realization *r* of the simulation:
        - Randomly select independent CDF values for load and resistance constructed imprecise probability structures and compute the corresponding interval load $\tilde{S}$ and interval resistance $\tilde{R}$ for the selected CDF values.
        - Determine and store interval bounds on the uncertain performance function.

    c. Repeat sufficiently large number of realizations to construct imprecise probability structure for uncertain performance function.
3. Determine the interval probability of failure for each failure mode by computing the performance function at the performance limit for each bound of the corresponding imprecise probability structure.
4. Determine the interval probability of failure of the structure using obtained intervals of probability of failure for each mode for two extreme cases of perfect correlation and independence among the modes as:

$$\max\left[\min(P_f^1), \min(P_f^2), ..., \min(P_f^m)\right] \le P_f \le 1 - \prod_{j=1}^{m}\left[1 - \max(P_f^j)\right] \quad (13)$$

5. Determine the maximum probability of failure as the upper bound of the interval probability of failure of the structure (independence case).

4. Example

In this section, an illustrative example is provided to demonstrate the applicability of the developed method in investigating the structural condition of a timber pedestrian bridge, shown in Figure 1. An increase in the bridge's live load (pedestrians using the bridge) in recent months has raised a concern over its safety.

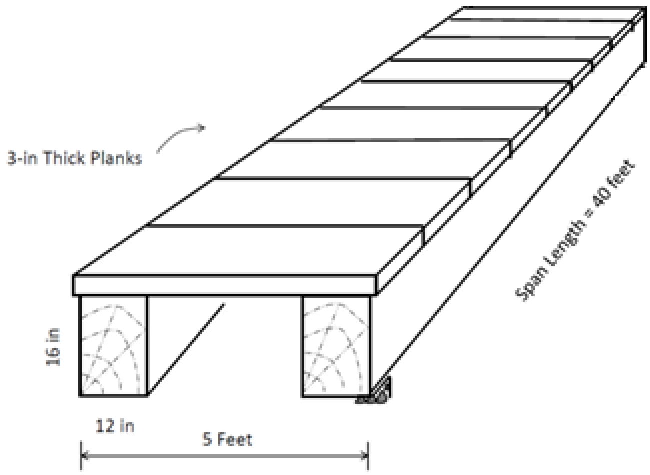

Figure 1. Schematics of a pedestrian bridge in the example.

4.1. PROBLEM PARAMETERS

The dominant modes of failure, as considered in this analysis, include bending, shear, and deflection modes of failure for the two beams. The input parameters are explained in detail in the authors' pervious work (Mohammadi and Modares, 2013). Based on the results from the previous analysis, Table I summarizes the probabilistic values (using Gaussian distributions) of the mean, μ , and standard deviation, σ , of the resistance and load for each failure mode.

Table I. Probabilistic values for load and resistance for considered failure modes.

Mode	Resistance		Load	
Bending	Bending Capacity		Applied Bending Stress	
	μ_R^b (psi)	σ_R^b (psi)	μ_S^b (psi)	σ_S^b (psi)
	6,990	1,820	1,730	416
Shear	Shear Capacity		Applied Shear Stress	
	μ_R^s (psi)	σ_R^s (psi)	μ_S^s (psi)	σ_S^s (psi)
	825	214	57.8	13.9
Deflection	Deflection Limit		Induced Deflection	
	μ_R^d (in)	σ_R^d (in)	μ_S^d (in)	σ_S^d (in)
	4.0	0	2.41	1.01

4.2. TRADITIONAL PROBABILITY ANALYSIS

The formulation for traditional probability analysis is used to compute the probability of failure for each failure mode (Eq. (11)). The results are summarized in Table II.

Table II. Probability of failure for each failure mode.

Mode	Probability of Failure, P_f
Bending	2.40×10^{-3}
Shear	1.73×10^{-4}
Deflection	5.77×10^{-2}

Considering the two extreme cases of perfect correlation and independence among the failure modes, the probability of failure for the structure can be bounded as (Eq. (12)):

$$0.0577 \le P_f \le 0.0602$$

In order to ascertain the reliability of the structure, the upper bound (independence case) can be used for the reliability level of the structure, $P_f = 0.0602$. It is worth noting that traditional probability analysis methods (including FORM) are not capable of considering uncertainties and variations in the mean or standard deviation of either load or resistance. The framework of imprecise probability structures allows for consideration of these uncertainties as depicted in the alternate solution of this example problem.

4.3. Imprecise Probability Structural Condition Assessment Analysis

The example problem is reanalyzed considering resistance and load values defined by imprecise probability structures for both the bending and shear failure modes. As the resistance value given in Table I for the deflection mode of failure is a code limit, it is not a random value, and thus only a deterministic CDF was used to model its value. An imprecise probability structure was also used to model the load (induced deflection) for the deflection mode of failure. All imprecise probability structures were generated by considering a ±10% shift in the mean values defining each random variable. Figures 2-4 depict the imprecise probability structures for load and resistance of bending, shear, and deflection modes, respectively (except for the resistance for the deflection failure mode).

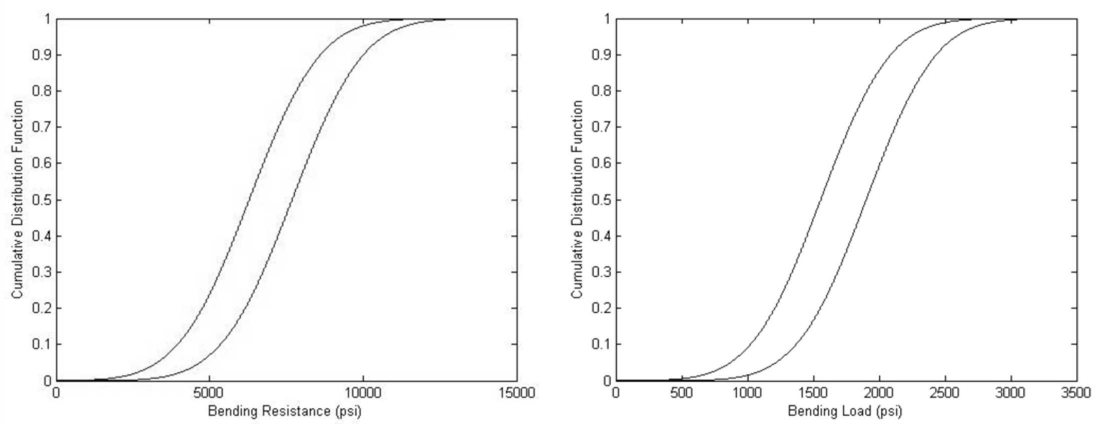

Figure 2. Imprecise probability structures for the resistance and load in bending mode.

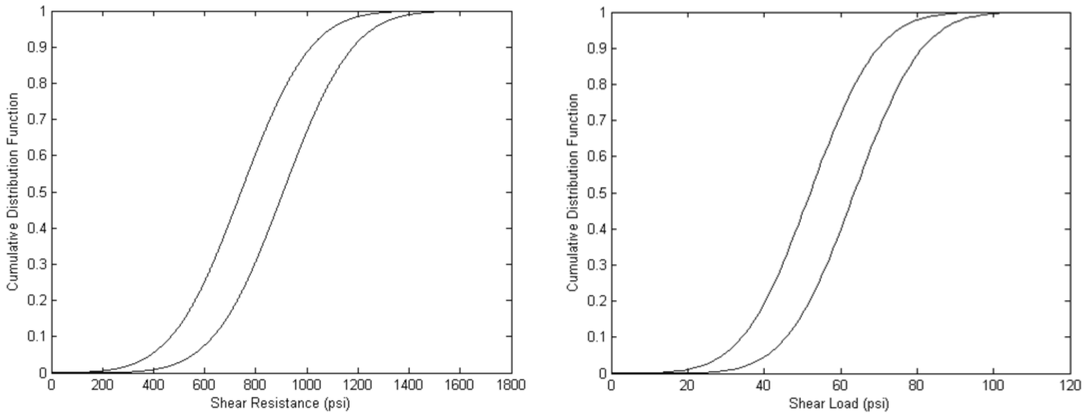

Figure 3. Imprecise probability structures for the resistance and load in shear mode.

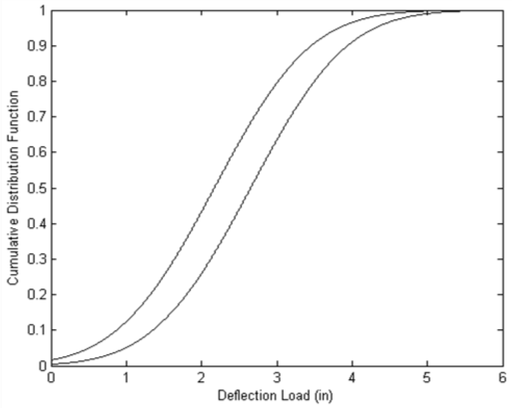

Figure 4. Imprecise probability structure for the induced deflection in the deflection mode.

4.4. SOLUTION

The Imprecise Probability Structural Condition Assessment (IPSCA) methodology was utilized for computing interval bounds on the probability of failure for each failure mode. These interval probabilities of failure were then used to compute an interval bound on the probability of failure of the structure. One million Monte Carlo realizations are performed and the interval bounds on the probability of failure for each failure mode and for the structure are determined (Table III).

Table III. Interval bounds on the probability of failure for each failure mode and for the structure.

Bending		*Shear*		*Deflection*		***Structure***	
Lower Bound	*Upper Bound*	*Lower Bound*	*Upper Bound*	*Lower Bound*	*Upper Bound*	***Lower Bound***	***Upper Bound***
4.76E-04	9.36E-03	3.00E-05	6.98E-04	3.50E-02	9.12E-02	**3.50E-02**	**1.00E-01**

4.5. Observations

As shown in Table III, the probability of failure of the structure is dominated by the single failure mode with the greatest probability of failure. Moreover, the results determined using IPSCA contain the FORM results, verifying the developed method.

5. Summary and Conclusions

In this work, a new method for reliability analysis of a structure using an imprecise probability approach is developed. This method, entitled Imprecise Probability Structural Condition Assessment (IPSCA), offers a new direction for incorporating uncertainties in condition assessment of structural systems. As IPSCA does not place restrictive assumptions typical in traditional probabilistic structural condition assessment methods, it provides a more realistic and comprehensive yet simpler process of treating uncertainties than traditional probabilistic-based reliability analyses. This method allows for uncertainty in the load and resistance for each mode of failure using imprecise probability structures. An example problem illustrating the application of the developed method demonstrated the application and computational feasibility of IPSCA. The simplicity of the proposed method makes it attractive for introducing uncertainty defined by imprecise probability into structural condition assessment procedures.

References

Ang, A. H.-S. and W. H. Tang. "Probability Concept in Engineering: Emphasis on applications to Civil and Environmental Engineering" Wiley, Hoboken, NJ., 2007.

Augenbaugh, J. M. and C. J. J. Paredis. "The Value of Using Imprecise Probabilities in Engineering Design." Journal of Mechanical Design, 128(4):969-979, 2006.

Ferson, S., V. Kreinovich, L. Ginzburg, D. S. Myers et al. Construction probability boxes and Dempster-Shafer structures, Tech. Rep. SAND2002-4015. Sandia National Laboratory, 2003.

Modares, M., R. L. Mullen and R. L. Muhanna. "Natural Frequencies of a Structure with Bounded Uncertainty." Journal of Engineering Mechanics, Dec. Ed., 1363-1371, 2006.

Mohammadi, J. and M. Modares. "Practical Approach to Using Uncertainties in Structural Condition Assessment." Practice Periodical on Structural Design and Construction, 18:155-164, 2013.

Muhanna, R. L. and R. L. Mullen. "Uncertainty in Mechanics Problems-Interval-Based Approach." Journal of Engineering Mechanics, June Ed., 557-566, 2001.

Oberkampf, W., J. Helton and K. Sentz. "Mathematical Representations of Uncertainty." AIAA Non-Deterministic Approaches Forum, AIAA 2001-1645. Seattle, 2001.

Williamson, R. and T. Downs. "Probabilistic Arithmetic I: Numerical Methods for Calculating Convolutions and Dependency Bounds." International Journal of Approximate Reasoning, 4:89-158, 1990.

Zhang, H., R. L. Mullen and R. L. Muhanna. "Finite Element Structural Analysis using Imprecise Probabilities Based on P-Box Representations." 4th International Workshop on Reliable Engineering Computing. Singapore, 211-225, 2010.